

Exercise 10

1. Calculate the conjugate of the following functions:

(a) Negative logarithm: $f(x) = -\sum_{i=1}^N \log x_i$;

(b) Quadratic function: $f(x) = x^T A x + b^T x + c$, where $A \in \mathbb{R}^{N \times N}$ is a symmetric positive definite matrix.

(c) Norm: $f(x) = \|x\|$, $x \in \mathbb{R}^N$.

2. Consider Assuming $a_i > 0, p \geq 1$, use KKT condition to give the solution of the problem

$$\min_{x \in \mathbb{R}^N} \left\{ \sum_i \frac{a_i}{x_i} : x_i > 0, \sum_i x_i^p \leq 1 \right\}.$$

3. Let $f(x) = x \log x$ with $\text{dom } f = \mathbb{R}_+$ and $f(0) = a$. Determine whether f is a closed function.